1. Surface area of a box.

The surface area is $\boxed{17\ \text{cm}^2}$. Let the dimensions of the box be x, y and z cm. Then we are given that $x^2 + y^2 + z^2 = 8^2 = 64$, and $4(x + y + z) = 36$. From this we calculate

$$81 = 9^2 = (x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + xz) = 64 + 2(xy + yz + xz).$$

Then the surface area is

$$2(xy + yz + xz) = 81 - 64 = 17\ \text{cm}^2.$$

2. Minimum value of a sequence

The minimum value is $\boxed{T_{129} = 7/4}$. From $t_1 = 98$ and $t_{13} = 89$ we calculate the common difference to be $-3/4$, so that in general $t_n = 98 - \frac{3}{4}(n - 1)$. Then

$$\begin{aligned} T_n &= 7 \cdot 98 - \frac{3}{4}[(n-1) + n + (n+1) + \cdots + (n+5)] \\ &= 7 \cdot 98 - \frac{3}{4}(7n + 14) \\ &= \frac{7}{4}(386 - 3n). \end{aligned}$$

This is itself an arithmetic progression, with common difference $-21/4$, and we are looking for the term nearest to 0. We find $T_{128} = 7/2$ and $T_{129} = -7/4$, so $|T_n|$ has minimum value $7/4$, occurring at $n = 129$.

3. Counting rational numbers.

There are $\boxed{64}$ such fractions. The integer 17! is a product of the form $2^{e_1}3^{e_2}5^{e_3}7^{e_4}11^{e_5}13^{e_6}17^{e_7}$; i.e., a product of seven prime powers $p_k^{e_k}$. For a fraction to be in lowest terms the numerator and denominator cannot have any common prime factors. Thus for each prime power $p_k^{e_k}$, the entire quantity $p_k^{e_k}$ must be either in the numerator or the denominator. There are 2^7 ways to choose prime powers for the numerator, leaving in each case the remaining prime powers in the denominator. In half of these the numerator is the smaller of the two products, so there are $2^6 = 64$ rational numbers of the desired kind.

4. Sets with sum 2015.

There are seven such sets, as we shall show. We seek integers n and k with $n \geq 2$ and $k \geq 0$ satisfying

$$2015 = (k+1) + (k+2) + \cdots + (k+n) = \frac{n(2k+n+1)}{2};$$

i.e., $2 \cdot 2015 = 2 \cdot 5 \cdot 13 \cdot 31 = n(2k+n+1)$. As $2 \leq n < 2k+n+1$, the possible values for n are 2, 5, 10, 13, 26, 31 and 62, giving the following seven solutions:

$$n = 2,\ 2k+n+1 = 2k+3 = 2015;\ 2k = 2012,\ k = 1006.\ \boxed{2015{=}1007{+}1008.}$$

$$n = 5,\ 2k+6 = 806;\ 2k = 800;\ k = 400.\ \boxed{2015{=}401{+}402{+}403{+}404{+}405.}$$

$$n = 10,\ 2k+11 = 403;\ k = 196.\ \boxed{2015 = 197 + 198 + \cdots + 206.}$$

$$n = 13,\ 2k+14 = 310;\ k = 148.\ \boxed{2015 = 149 + 150 + \cdots + 162.}$$

$$n = 26,\ 2k+27 = 155;\ k = 64.\ \boxed{2015 = 65 + 66 + \cdots + 90.}$$

$$n = 31,\ 2k+32 = 130;\ k = 49.\ \boxed{2015 = 50 + 51 + \cdots + 80.}$$

$$n = 62,\ 2k+63 = 65;\ k = 1.\ \boxed{2015 = 2 + 3 + \cdots + 63.}$$

5. Divisibility.

The values are $\boxed{n = -8,\ n = 0 \text{ and } n = 2}$. By division we find

$$n^3 - 8n^2 + 2n = (n^2+1)(n-8) + (n+8),$$

so $n^3 - 8n^2 + 2n$ is divisible by n^2+1 if and only if $n+8$ is divisible by n^2+1. This is the case if $n = -8$ or 0 or 2. To see that there are no other values we observe that if $n + 8 \neq 0$ we need $n^2 + 1 \leq |n+8|$.

Case 1: $n^2 + 1 \leq n + 8$. This is equivalent to

$$\left(n - \frac{1}{2}\right)^2 \leq \frac{29}{4}; \quad \text{i.e.,} \quad -\frac{\sqrt{29}}{2} \leq \left(n - \frac{1}{2}\right) \leq \frac{\sqrt{29}}{2}.$$

Thus

$$-4 < -\frac{\sqrt{29}}{2} + \frac{1}{2} \leq n \leq \frac{\sqrt{29}}{2} + \frac{1}{2} < 4.$$

In this range, 0 and 2 are the only integers satisfying the divisibility condition.

Case 2: $n^2 + 1 \leq -n - 8$. This is equivalent to

$$\left(n + \frac{1}{2}\right)^2 = n^2 + n + \frac{1}{4} \leq -\frac{35}{4},$$

which is impossible for real n. Thus the only solutions are -8, 0 and 2.

6. Limit of a sequence.

The unique solution is $\boxed{c = e - 1}$. Let $f(x) = 1/(1 + cx)$, $1 \le x \le 1$. Given $n > 0$, let $x_k = k/n$ for $k = 1$ to n, and $\Delta x_k = 1/n$. The corresponding Riemann sum for

$$\int_0^1 f(x)dx \qquad \text{is} \qquad \sum_{k=1}^{n} f(x_k)\Delta x_k = \sum_{k=1}^{n} \frac{1}{1 + ck/n} \cdot \frac{1}{n} = \sum_{k=1}^{n} \frac{1}{n + kc}.$$

Then

$$\lim_{n\to\infty} \sum_{k=1}^{n} \frac{1}{n + kc} = \int_0^1 \frac{dx}{1 + cx} = \frac{1}{c} \ln(1 + cx)|_0^1 = \frac{1}{c} \ln(1 + c),$$

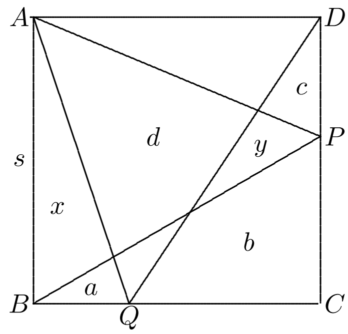
which is equal to $1/c$ if and only if $\ln(1 + c) = 1$; i.e., $1 + c = e$; i.e.,$c = e - 1$.

7. n^{2015} as a sum of squares.

Let p and q be any positive integers, and $a = (p^2 + q^2)^{1007}p$, $b = (p^2 + q^2)^{1007}q$. Then

$$a^2 + b^2 = (p^2 + q^2)^{2014}p^2 + (p^2 + q^2)^{2014}q^2 = (p^2 + q^2)^{2014}(p^2 + q^2) = (p^2 + q^2)^{2015}.$$

8. Equal areas.



Let x and y be the triangular areas now indicated in the figure, and note that triangles ABP and AQD each have area $s^2/2$, where s is the side length of the square. Then

$$(a + x) + (b + y + c) = s^2 - |AQD| = s^2/2, \qquad (1)$$

and

$$d + x + y = |APB| = s^2/2. \qquad (2)$$

It follows from (1) and (2) that $a + b + c = d$.

9. A bound on the zeros of a polynomial.

If $|\alpha| \le 1$ then $|\alpha| < 7$, so assume $|\alpha| > 1$. From $P(\alpha) = 0$ we have

$$\begin{aligned} |\alpha|^5 &= |a_4\alpha^4 + a_3\alpha^3 + a_2\alpha^2 + a_1\alpha + a_0| \\ &\le |a_4||\alpha|^4 + |a_3||\alpha|^3 + |a_2||\alpha|^2 + |a_1||\alpha| + |a_0| \\ &\le 6(|\alpha|^4 + |\alpha|^3 + |\alpha|^2 + |\alpha| + 1) \\ &= 6\left(\frac{|\alpha|^5 - 1}{|\alpha| - 1}\right). \end{aligned}$$

This, with $|\alpha| > 1$, implies $|\alpha|^6 - |\alpha|^5 \le 6(|\alpha|^5 - 1) < 6|\alpha|^5$, whence $|\alpha|^6 < 7|\alpha|^5$ and therefore $|\alpha| < 7$.

10. Sum of squares less than 2015.

Let $m = -5$ and $M = 13$, and note that $-31mM = 2015$. Because $m \le a_j \le M$ for each j we have

$$\left(a_j - \frac{m+M}{2}\right)^2 \le \left(\frac{M-m}{2}\right)^2,$$

and therefore

$$\begin{aligned}
31\left(\frac{M-m}{2}\right)^2 &\ge \sum_{j=1}^{31}\left(a_j - \frac{m+M}{2}\right)^2 \\
&= \sum_{j=1}^{31} a_j^2 - (m+M)\sum_{j=1}^{31} a_j + 31\left(\frac{m+M}{2}\right)^2 \\
&= \sum_{j=1}^{31} a_j^2 + 31\left(\frac{m+M}{2}\right)^2.
\end{aligned}$$

Then

$$\sum_{j=1}^{31} a_j^2 \le 31\left[\left(\frac{M-m}{2}\right)^2 - \left(\frac{m+M}{2}\right)^2\right] = -31mM = (-31)(-5)(13) = 2015.$$