



# Mathematical Association of America

## North Central Section

Spring Meeting • April 5-6, 2024  
University of St. Thomas  
Minneapolis, Minnesota



<b>Friday, April 5 (6:00pm – 10:00pm)</b>	
6:00 – 8:30	<b>Registration</b> SCH 1 <sup>st</sup> Floor Atrium  \$25 for MAA Members, FREE for Students, Retired and VITAL Faculty
6:30 – 7:00	<b>Welcome and Announcements</b> <b>Welcome by Dr. Magda Stolarska, Chair, Department of Mathematics</b> SCH 120  Jeremiah Bartz, MAA-NCS President
7:00 – 7:50	<b>Estimation Game</b> SCH 120  Led by: Rob Thompson, Carleton College Kunwu Lyu, Carleton College Jacob Aronson, Carleton College
8:00 – 9:00	<b>A Friendly Introduction to Graph Signal Processing</b> SCH 120 Jeremiah Bartz, Presiding  Kris Hollingsworth, MAA-NCS Section NExT Minnesota State University Mankato
9:00 – 10:00	<b>Reception</b> SCH 2 <sup>nd</sup> Floor Atrium

*Thank you to University of St. Thomas's faculty, staff and students involved in the effort of hosting this meeting.*

## Saturday, April 6 (7:30am – 4:30pm)

7:30 – 11:00	<b>Registration and Breakfast</b> SCH 1 <sup>st</sup> Floor Atrium and SCH 2 <sup>nd</sup> Floor Atrium	
7:40 – 8:20	<b>Business Meeting</b> SCH 120  <div style="text-align: right;">Jeremiah Bartz, MAA-NCS President</div>	
8:20 – 8:30	<b>Welcome by Dr. William Tolman, Dean, College of Arts and Sciences</b> <b>University of St. Thomas</b> SCH 120	
8:30 – 9:20	<b>The Mathematics of Secrets</b> SCH 120 Jeremiah Bartz, Presiding  <div style="text-align: right;">Adriana Salerno, MAA Section Visitor Bates College</div>	
	<b>SCH 301</b>	<b>SCH 302</b>
9:30 – 9:50	<b>Overpartitions with Only Even Parts Overlined</b>  <div style="text-align: center;">Aidan Carlson, University of Minnesota Duluth James Sellers, University of Minnesota Duluth</div>	<b>Hidden Gems of Numerical Mathematics</b>  <div style="text-align: center;">Pavel Bělík, Augsburg University</div>
9:55 – 10:15	<b>When are Satellite Orbits Stable?</b>  <div style="text-align: center;">Ritwik Gaur, UM/Wayzata High School Jodin Morey, University of Minnesota</div>	<b>Cardano Struggles with Homogeneity</b>  <div style="text-align: center;">William Branson, St. Cloud State University</div>
10:20 – 10:40	<b>BatMath! An Integrodifference Equation Model of Metapopulations Illuminates the Effect of Climate Change on Equilibria</b>  <div style="text-align: center;">Peter Roggenbuck, Winona State University Joseph Knopp, Winona State University</div>	<b>Experience and Results of PLAINS REU Site</b>  <div style="text-align: center;">Jung-Han Kimn, South Dakota State University Stephen Gent, South Dakota State University Tyler Miller, South Dakota State University</div>
10:40 – 10:50	<b>Break</b>	
10:50 – 11:10	<b>Scoring Estimation Games</b>  <div style="text-align: center;">Jacob Aronson, Carleton College Kunwu Lyu, Carleton College</div>	<b>Elementary Proofs of Congruences for POND and PEND Partitions</b>  <div style="text-align: center;">James Sellers, University of Minnesota Duluth</div>
11:15 – 11:35	<b>Plato Encounters a Glass Sealing</b>  <div style="text-align: center;">Tom Sibley, CSBSJU Genevieve Ahlstrom, University of St. Thomas</div>	<b>Hooked on Arrowgrams</b>  <div style="text-align: center;">Ken Price, University of Wisconsin Oshkosh</div>
11:40 – 12:00	<b>A Network Science Approach to Fungal Networks and the “Wood Wide Web”</b>  <div style="text-align: center;">Paula Mercurio, St. Olaf College Emelyce Bigirimana, St. Olaf College Kaashya Khandelwal, St. Olaf College</div>	<b>Computational Math: A Way of Thinking</b>  <div style="text-align: center;">Matthew Wright, St. Olaf College</div>

## Saturday, April 6 (7:30am – 4:30pm)

12:00 – 1:15	<b>Lunch</b>	
1:15 – 2:15	<p><b>Panel: Mathematics and Data for Social Justice – Community and Teaching</b> SCH 120</p> <p style="text-align: right;">John Zobitz, Augsburg University Julia Walk, Concordia College Moorhead Aaron Wangberg, Winona State University Megan Breit-Goodwin, Anoka-Ramsey Community College</p>	
	<b>SCH 301</b>	<b>SCH 302</b>
2:25 – 2:45	<p><b>Adiabatic Gluing of Floer Trajectories and Applications</b></p> <p style="text-align: right;">Ke Zhu, Minnesota State University Mankato Yong-Geun Oh, IBS Center for Geometry and Physics</p>	<p><b>Congruent Triangles (Almost Successful Activities)</b></p> <p style="text-align: right;">Mike Weimerskirch, University of Minnesota Shelley Kandola, University of Minnesota</p>
2:50 – 3:10	<p><b>The Universal Lipschitz Path Space of a Purely 2-Unrectifiable Space</b></p> <p style="text-align: right;">Daniel Perry, Augustana University</p>	<p><b>Simple Framework for Students Teaching Each Other Proof Strategies (Almost Successful Activities)</b></p> <p style="text-align: right;">Jed Yang, Bethel University</p>
3:15 – 3:35	<p><b>Games with Permutation Groups: Move and Return</b></p> <p style="text-align: right;">Bret Benesh, CSBSJU</p>	<p><b>Hardy's Apology (Almost Successful Activities)</b></p> <p style="text-align: right;">Jeff Ford, Gustavus Adolphus College</p>
3:35 – 3:45	<b>Break</b>	
3:45 – 4:05	<p><b>ODEs and Mandatory Voting</b></p> <p style="text-align: right;">Natasa Dragovic, University of St. Thomas Christoph Borgers, Tufts University Anna Haensch, Tufts University Arkadz Kirshtein, Tufts University Lilla Orr, University of Richmond</p>	<p><b>A Friendly Introduction to Abstract Algebra</b></p> <p style="text-align: right;">Ryota Matsuura, St. Olaf College</p>
4:10 – 4:30	<p><b>How I Learned to Love Rational Approximation</b></p> <p style="text-align: right;">Will Mitchell, Macalester College Johan Azambou, Macalester College</p>	<p><b>A Transition-to-Higher-Mathematics Course as an Environment where Mathematics and Computer Science Inform Each Other</b></p> <p style="text-align: right;">Stephen M. Walk, St. Cloud State University</p>

# Abstracts

## Invited Addresses

**Kris Hollingsworth, Minnesota State University Mankato &  
MAA-NCS Section NExT**  
A Friendly Introduction to Graph Signal Processing

Over the past decade, researchers in Mathematics and Electrical Engineering have been interested in generalizing techniques for signal processing on regular, Euclidean domains (such as  $\mathbb{R}^n$  or  $\mathbb{C}^n$ ) to irregular domains such as graphs. Researchers believe that by incorporating the geometric structure of the graph, we will obtain better signal processing techniques for applications. In this talk, I will discuss motivations for the proposed definitions for the graph Fourier transform and introduce several challenges which arise in generalizing traditional Fourier analysis techniques to irregular domains. If time permits, I will introduce some results which are joint work with Mahya Ghandehari and Dominique Guillot. No background knowledge will be expected or required beyond basic Calculus and Linear Algebra.



**Adriana Salerno, Bates College &  
MAA Section Visitor**  
The Mathematics of Secrets

Information permeates our lives. We send texts, shop online, pay bills, type emails, and store pictures and medical data in “the cloud”. Hidden from view is the mathematics needed to make information transfer efficient and secure. This talk will give an overview of the history and the evolution of coding (turning information into numbers) and encryption (securing the information), from ancient times to the modern era, and we will look at how mathematics is the key to it all.



## Panel

### **Mathematics and Data for Social Justice – Community and Teaching**

Speakers:      John Zobitz, Augsburg University  
                      Julia Walk, Concordia College Moorhead  
                      Aaron Wangberg, Winona State University  
                      Megan Breit-Goodwin, Anoka-Ramsey Community College

The MAA-NCS hosted a summer seminar on Mathematics and Data for Social Justice in summer 2023. Join us for a conversation about the seminar, and about the ways mathematics curriculum and teaching and learning can be spaces for social justice development. We invite you to join us in ongoing community around this topic within the section.

# Contributed Talks

**Jacob Aronson, Carleton College, Kunwu Lyu, Carleton College**

Scoring Estimation Games

What is the dollar value of one US ton of pennies? How many people visited the Minnesota State Fair in 2023? In an estimation game, participants respond to questions like these by providing a numerical interval they believe contains the true answer. Participants receive a score for their response. How should this score be calculated? For example, should the score for a large interval which contains the true answer be better or worse than the score for a small interval which is near to (but does not contain) the true answer? In this talk, we'll discuss our approach to scoring (and running) estimation games in a fun and fair way.

**Pavel Bělík, Augsburg University**

Hidden Gems of Numerical Mathematics

In this talk I will visit a few concepts in numerical mathematics that are less known, not often mentioned in numerical analysis/mathematics books or numerical analysis classes, yet have a surprising power or ability to solve various problems.

**Bret Benesh, College of Saint Benedict and Saint John's University**

Games with Permutation Groups: Move and Return

The game Return is played on a square with corners numbered 1—4 and a marker set by Corner 1. The first player selects any of the eight symmetries, and the marker moves accordingly. The second player selects one of the seven remaining symmetries and the marker accordingly. Play continues until there are no symmetries remaining. The first player wins exactly when the marker is at Corner 1 at the end of the game. We give full results for all regular polygons, study a similar game called Move, and give some results for the generalized games on permutation groups.

**William Branson, St. Cloud State University**

Cardano Struggles with Homogeneity

In Chapter 7 of his 1545 text *Ars Magna*, Girolamo Cardano presented a correct rule for transforming the equation  $x^3 + N = ax$  into  $y^3 + M = ay^2$ . He also wanted a geometric demonstration of this rule, but ran into a problem: what is the dimension of  $a$ ? This was not Cardano's only attempt to represent algebraic equations in geometric demonstrations; this talk will examine several other examples from the *Ars Magna* and will place them in the history of the development of algebraic proofs.

**Aidan Carlson (UG), University of Minnesota Duluth, James Sellers, University of Minnesota Duluth**

Overpartitions with Only Even Parts Overlined

A partition of an integer  $n$  is a representation of  $n$  as the sum of one or more positive integers, or parts, where the parts are listed in non-increasing order. The number of such partitions of  $n$  is denoted by  $p(n)$ . For example, the 5 integer partitions of  $n = 4$  are

$$(4), (3 + 1), (2 + 2), (2 + 1 + 1), (1 + 1 + 1 + 1).$$

From that example, we see that  $p(4) = 5$ .

Stemming from this idea, a different type of partition was created. These new partitions are called *overpartitions*. An overpartition is a partition where the first instance of each part can be "overlined". We denote the function which counts these kinds of partitions by  $\bar{p}(n)$ . For example,  $\bar{p}(3) = 8$  because we have the following overpartitions which add up to 3:

$$(3), (\bar{3}), (2 + 1), (\bar{2} + 1), (2 + \bar{1}), (\bar{2} + \bar{1}), (1 + 1 + 1), (\bar{1} + 1 + 1).$$

In this talk, we will explore the foundations of these two topics and the insights and advancements made by Euler and Ramanujan. Taking these ideas and expanding on a paper written by Augustine Munagi and James Sellers in

2014, we will explore the overpartition function  $D_2(n)$  and show some of its congruences modulo 2 and modulo 4. (The function  $D_2(n)$  counts the number of overpartitions of  $n$  where only even parts can be overlined.)

**Natasa Dragovic, University of St. Thomas**, Christoph Borgers, Tufts University, Anna Haensch, Tufts University, Arkadz Kirshtein, Tufts University, Lilla Orr, University of Richmond  
ODEs and Mandatory Voting

In this talk we will discuss mathematics relevant to the question whether voting should be mandatory. Assuming a static distribution of voters' political beliefs, we model how politicians might adjust their positions to raise their share of the vote. Various scenarios can be explored using our app at <https://centrism.streamlit.app/>. Abstentions are found to have great impact on the dynamics of candidates, and in particular to introduce the possibility of discontinuous jumps in optimal candidate positions. This is an unusual application of ODEs. We hope that it might help engage some students who may find it harder to connect with the more customary applications from the natural sciences.

**Jeff Ford, Gustavus Adolphus College**  
Hardy's Apology

I attempted to show my students how the work of G.H. Hardy went from mathematics for its own sake to something with modern applications in cryptography. I pitched this as the completely wrong level for the course. With enough modifications the students came to appreciate the concepts, but the level of inquiry needed was far too compressed in my original construction. (For MITN's Almost Successful Activities Session)

**Ritwik Gaur (HS), University of Minnesota/Wayzata High School**, Jodin Morey, University of Minnesota  
When are Satellite Orbits Stable?

In this talk, we delve into the captivating field of dynamical systems by examining the behavior of satellites in a gravitational field. We investigate what happens to a pinwheel-point mass configuration (representing a satellite orbiting a body in the universe) along its orbit. Join us as we find which configurations lead to equilibria and uncover criteria for stable orbits.

**Jung-Han Kimn, South Dakota State University**, Stephen Gent, South Dakota State University, Tyler Miller, South Dakota State University  
Experience and Results of PLAINS REU Site

South Dakota State University PLAINS (Promoting Leadership in Advanced research computing for Interdisciplinary Sectors) REU program provide students technical tools to become effective researchers and holistic experience for learning how to conduct effective and responsible research in a variety of STEM areas including engineering, mathematical sciences, and biological sciences disciplines. We will present the results and lesson we learned from our REU site experiences.

**Ryota Matsuura, St. Olaf College**  
A Friendly Introduction to Abstract Algebra

We will describe an innovative approach to laying a foundation for abstract mathematics. When students generalize from a wide range of examples, they are better equipped to conjecture, formalize, and prove new ideas. Thus, they should explore concepts through illuminating examples before formal definitions/theorems are introduced. Rather than merely consuming mathematical knowledge, students should learn mathematics by actively creating mathematics.

Abstract algebra often acts as "gateway" to completing a mathematics major. But it can seem impenetrable due to its (seemingly) theoretic nature. By taking a more concrete approach and allowing students to develop their own understanding, we can make abstract algebra more accessible to more students.

**Paula Mercurio, St. Olaf College**, Emelyce Bigirimana, St. Olaf College, Kaashya Khandelwal, St. Olaf College  
A Network Science Approach to Fungal Networks and the “Wood Wide Web”

As they grow, fungi naturally form underground networks from the growth of hyphae, fungal “roots” that divide and split off from each other, forming large, interconnected networks over time. These networks can interact with the roots of plants, creating an underground resource-sharing network connecting fungi to plants and trees. In this talk, I will discuss how we can use tools from graph theory and network science to model and investigate the relationships between the plants and fungi in these networks and discuss how network models may give us some insight into the ecological role of these underground connections.

**Will Mitchell, Macalester College**, Johan Azambou, Macalester College  
How I Learned to Love Rational Approximation

In 1812, Gauss described a method for finding rational functions to approximate  $\phi(z) = \log \frac{z+1}{z-1}$ . Gauss was interested in this problem for a surprising reason: any strategy for numerical integration on a finite interval yields a rational approximation of  $\phi$ , and vice versa. I will describe this connection and give some results on a project to apply recent advances in rational approximation (especially Trefethen's recent AAA algorithm) to the evaluation of challenging definite integrals.

**Daniel Perry, Augustana University**  
The Universal Lipschitz Path Space of a Purely 2-Unrectifiable Space

The goal of this talk is to define and inspect a metric version of the universal path space and study its application to purely 2-unrectifiable spaces, in particular the Heisenberg group. The construction of the universal Lipschitz path space, as the metric version is called, echoes the classical construction of the universal cover in algebraic topology. Similar to the universal cover, the universal Lipschitz path space of a purely 2-unrectifiable space satisfies a unique lifting property, a universal property, and is Lipschitz simply connected. Applications to the study of Lipschitz fundamental groups will be discussed.

**Ken Price, University of Wisconsin Oshkosh**  
Hooked on Arrowgrams

Take a deep dive into the fascinating realm of Arrowgrams and uncover the hidden messages concealed within directed graphs. These captivating puzzles start with labels on some arrows, challenging solvers to label the rest using the transitive relation. Dr. Price will provide motivation for studying Arrowgrams, demonstrate how to create them using student-authored software, and hint at potential undergraduate research-projects. Prepare to be hooked not only on the joy of solving Arrowgrams, but also by the intriguing mathematical ideas behind them.

**Peter Roggenbuck, Winona State University, Joseph Knopp (UG), Winona State University**  
BatMath! An Integrodifference Equation Model of Metapopulations Illuminates the Effect of Climate Change on Equilibria

In this talk we develop an integrodifference equation model of metapopulations in point-patch habitats (roosts). The model implements forage and homing features to track changes in populations among roosts. This model is appropriate for a wide variety of organisms whose colonies are located on isolated landscape points such as bat caves, bee hives, and ant colonies. We use a separation of scales method to derive analytic approximations of equilibria for each roost. These approximations allow for an analysis of the effects of climate change on the long-term state of a metapopulation. To parameterize the model, we developed a simulated environment using an agent-based modeling framework (NetLogo) to generate population data.

**James Sellers, University of Minnesota Duluth**

Elementary Proofs of Congruences for POND and PEND Partitions

Recently, Ballantine and Welch considered two classes of integer partitions which they labeled POND and PEND partitions. These are integer partitions wherein the odd parts (respectively, the even parts) cannot be distinct. In recent work, I studied these two types of partitions from an arithmetic perspective and proved infinite families of mod 3 congruences satisfied by the two corresponding enumerating functions. I will talk about the generating functions for these enumerating functions, and I will also highlight the (induction) proofs that I utilized. The talk will be self-contained and should be accessible to a wide audience.

**Tom Sibley, St. John's University and College of St. Benedict (retired), Genevieve Ahlstrom (G), University of St. Thomas**

Plato Encounters a Glass Sealing

There are lovely interactions of the shapes and symmetries of the platonic solids. We sought to embody these using stained glass to craft four sculptures of regular polyhedra inscribed in other regular polyhedra. We hope you can see the platonic ideal in spite of the physical challenges we encountered with the media of stained glass and solder.

**Stephen M. Walk, St. Cloud State University**

A Transition-to-Higher-Mathematics Course as an Environment Where Mathematics and Computer Science Inform Each Other

Our "transition" course is for students of mathematics and students of computer science and software engineering. It includes some concepts that lean one way or the other, appearing either "more specific to computing" or "more math-y." But even the topics that appear to fall squarely into one category can contribute to student understanding of the topics in the other category. In this talk, we will explore connections going in both directions: mathematical ideas that illuminate computer science concepts and computer science ideas that illuminate mathematical concepts.

**Mike Weimerskirch, University of Minnesota, Shelley Kandola, University of Minnesota**

Congruent Triangles

This activity is an introduction to the Law of Sines/Law of Cosines and Solving Triangles. (For MITN's Almost Successful Activities Session)

**Matthew Wright, St. Olaf College**

Computational Mathematics: A Way of Thinking

Computational skills are undeniably important for math majors, yet the development of such skills is often overlooked. In particular, many students lack the ability to take an open-ended math question, design and run computational experiments (in a computer algebra system or programming environment), and formulate conjectures based on their observations. In this talk, I will present some ideas for filling the gap in students' computational abilities, with examples from the Modern Computational Mathematics course at St. Olaf College. This course helps students develop computational fluency, learn to ask good questions, and gain valuable transferable skills.

**Jed Yang, Bethel University**

Simple Framework for Students Teaching Each Other Proof Strategies

In our Transition to Proofs course taught in a flipped format, students do preview exercises as homework and present some to each other in class. On one particular day, students present various proof strategies (direct proof, indirect proof, and proof by contradiction), before spending the rest of the day working through exercises based on these and other proof strategies. The day's success is, thus, somewhat dependent on the student presentations. We will discuss some ways to reduce such dependency while keeping the benefits of this framework. (For MITN's Almost Successful Activities Session)



**Ke Zhu, Minnesota State University Mankato**, Yong-Geun Oh, IBS Center for Geometry and Physics  
Adiabatic Gluing of Floer Trajectories and Applications

We study the adiabatic degeneration of Floer trajectories to “disk-flow-disk” configurations and the recovering gluing, where the gradient flow part has positive length. Unlike the standard gluing problem, we study the problem of gluing two objects of different dimensions: 1-dimensional gradient segments and 2-dimensional (perturbed) J-holomorphic maps. Similar configurations also appear in the Morse-Bott approach for symplectic homology and contact homology. As an immediate application, we outline the proof that when a finite number of Hamiltonian deformations of a monotone Lagrangian submanifold collapse simultaneously, the pearl complex moduli spaces by Biran-Cornea are diffeomorphic to the J-holomorphic polygon moduli spaces by Fukaya-Oh-Ono-Ohta, provided the dimension of the moduli spaces is sufficiently small e.g., when the dimension is 0,1 or 2. This is enough to prove that the  $A_\infty$ -structures appearing in the two pictures are isomorphic to each other. This also provides another proof of the isomorphism property of PSS map which is different from previous work of Oh-Zhu: It bypasses the nodal Floer trajectories by going directly from “disk-flow-disk” configurations to resolved Floer trajectories, without the need of blowing up target.

**NCS-MAA Fall 2024 Meeting** Augustana University, October 25-26