## North Central Section

## Mathematical Association of America



Fall Meeting • October 14-15, 2022<br>University of North Dakota<br>Grand Forks, North Dakota

Friday, October 14, 2022 (7:00PM - 10:00PM)
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\begin{array}{ll}\text { 7:00-8:30 } & \begin{array}{l}\text { Registration - Memorial Union 214 - CD Ballroom } \\
\$ 25 \text { (Free for Invited Speakers, Students, Retired, VITAL Faculty, HS Educators) }\end{array}
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Contributed Session - Memorial Union 220 - Small Ballroom Presiding: Doojin Hong\end{array}\right\}\)| 7:00-7:05 | Welcome: Bradley Rundquist, UND Dean of Arts and Sciences |
| :--- | :--- |
| $7: 05-7: 25$ | William Schwalm, University of North Dakota (retired), <br> Extending the Clairaut Type of Differential Equations to Higher Order |
| $7: 30-7: 50$ | Sarah Seger, Concordia College, <br> Closed Seifert Surfaces and 0.5-solvability |
| 7:55-8:15 | Jeremiah Bartz, University of North Dakota, <br> Visualizations of Balancing Number Identities |
| Invited Lecture - Memorial Union 220 - Small Ballroom |  |
| Presiding: Aaron Wangberg, MAA-NCS President |  |

Saturday, October 15, 2022 (8:30AM - 4:25PM)

| 8:30-11:00 | $\begin{array}{l}\text { Registration and Breakfast - Memorial Union 214 - D Ballroom } \\ \text { Invited Lecture -- Memorial Union 220 - Small Ballroom } \\ \text { Presiding: Aaron Wangberg, MAA-NCS President }\end{array}$ |
| :--- | :--- |
| 9:00-9:05 | Welcome: Andrew Armacost, UND President |$]$| 9:05-9:55 | Hortensia Soto, Colorado State and MAA President-Elect <br> Compassion in \& Access to Learning Mathematics (CALM) |
| :--- | :--- |
| Concurrent Session Ia - Memorial Union 220 - Small Ballroom Presiding: Doojin Hong |  |

$\left.\begin{array}{ll}1: 20-2: 10 & \begin{array}{l}\text { Business Meeting - Memorial Union 220 - Small Ballroom } \\ \text { Presiding: Aaron Wangberg, MAA-NCS President }\end{array} \\ \text { Concurrent Session IIa - Memorial Union 318 - AB Meeting Room Presiding: Gerri Dunnigan }\end{array}\right\}$

# Abstracts 

## Invited Addresses

Eric Egge, Carleton College,
Evidence-Based Teaching: A Case Study
Although I am not an expert on evidence-based teaching, the idea that we should systematically study our teaching methods, and favor using the ones that work, is appealing. So I recently tried to find out what is known about the effects, and effectiveness, of a specific teaching tactic. In this talk I'll share what I learned about this specific tactic, and about evidence-based teaching more generally.

Hotensia Soto, Colorado State University \& MAA President-Elect,<br>

Research indicates that students from minoritized groups are more likely to pursue STEM degrees if they can see how these fields benefit their communities and if they are in classrooms where they experience micro or macro-affirmations. In this presentation, I will share my perspectives, based on research and personal experiences, on how we can create learning environments that provide our students access to learning mathematics. I argue that we can help students see the value of mathematics by challenging them, providing a supportive learning environment, and creating a space where they have a voice in their learning.

## Panel

Building a Classroom Environment to Facilitate Student Access to \& Learning of Math<br>Moderator: Michele Iiams, University of North Dakota<br>Panelists:<br>Kristine Braaten, Turtle Mountain Community College<br>Semhar Michael, South Dakota State University<br>Cathy Williams, Bemidji State University

As a continuation of the keynote talk by Dr. Soto, members of the panel will share strategies for creating classroom environments that facilitate students' access to and learning of mathematics. Audience members will be encouraged to participate in this important conversation.

## Contributed Talks

Doug Anderson, Concordia College,<br>Isolated Time Scales, Periodicity, and Stability

Recently, a new definition of periodicity for functions defined on isolated time scales has been introduced. We will illustrate this for q -difference equations, and then use these periodic functions in simple first-order difference equations to explore questions of Ulam stability.

## Jeremiah Bartz, University of North Dakota,

Visualizations of Balancing Number Identities
The triple $(A, B, C)$ of positive integers is said to be a balancing triple if $T(A)+T(B)=T(C)$ where $T(k)=k(k+1) / 2$ is the $k$ th triangular number. For example, $(5,6,8)$ is a balancing triple since $T(5)+$ $T(6)=T(8)$. Due to their geometric origins, triangular numbers have natural visualizations. In this talk, we present visualizations of identities discovered while studying balancing numbers and their generalizations.

## Bryce Christopherson, University of North Dakota,

Expectation Thresholds and the Lottery Ticket Hypothesis
In 2019, Frankle and Carbin provided an interesting conjecture known as the lottery ticket hypothesis, which supposes that large random neural networks usually contain a 'lucky' sub-network that is 'just as good' as the larger network. That is, a typical large enough, dense, randomly initialized, feed-forward neural network can be pruned quite a great deal and this much sparser sub-network performs nearly identically to the original dense network after the weights of each are optimized in training. A stronger form of the conjecture due to Ramanujan et al. (that does away with even the necessity of training the sparse sub-network), was proven by Malach, Yehudai, et al. shortly thereafter in 2020, essentially showing that pruning allows for universal approximation in the same fashion as training. Around the same time--and in no relation to the lottery ticket hypothesis--Park and Pham proved the Kahn-Kalai conjecture: a bold claim regarding the point at which a phase transition occurs in a system, rendering some previously unlikely event likely. In this talk, we will discuss a connection between the Kahn-Kalai conjecture and the strong form of the lottery ticket hypothesis.

## Steven Collazos, University of Minnesota - Morris, Firing Rate Models and Convex Coding

Ever since at least D. Hebb, ensembles of neurons have been proposed as a basis for neural processing. One known model in that spirit is the framework of permitted sets. In this talk, we will discuss an extension to that framework and prove that the resulting neural code is convex in a low-rank regime.

## Jacob Denault (Grad), University of North Dakota, <br> A Pancake-Theorem Approach to Political Redistricting

The discrete ham sandwich theorem states that for a finite set of points in the plane, each colored "red" or "blue", there is a line that simultaneously bisects the red points and bisects the blue points. By assigning weights to the finite set of points in the plane, however, we will prove that the discrete ham sandwich theorem is but a special case of a more general theorem where the weights are balanced to within a welldefined tolerance.

Christopher Ennis, Normandale Community College (retired), John Sheir, Disjoint Placement Probability of Line Segments via Geometry

When any finite number $n$, of line segments having total combined length less than one, have their centers placed randomly inside the unit interval $[0,1]$, the probability of obtaining a mutually disjoint placement of the segments within $[0,1]$ is given by $(1-L)^{n}$ where $L=$ combined length of the segments. The result follows from the geometry of the event, "all segments disjoint and contained within $[0,1]$ ", considered as a subset of the uniform probability space of $n$ centers, each of which is in $[0,1]$; i.e. the unit $n$-cube. This event consists of $n$ ! disjoint, congruent, (up to a mirror image) polytopes within the unit $n$-cube, which fit together perfectly to form, except for a set of measure zero, a partition of an $n$ dimensional cube with common edge length $1-L$, and hence of $n$-volume $(1-L)^{n}$. These polytopes
are thus $n$-dimensional space filling. In the case of $n=3$, they form one of the known tetrahedral partitions of the cube.

## Hongyan Hou, Minnesota State University-Moorhead, Generalized Linear Models and Robust Statistics Regarding to the Normality Assumption in Linear Regression

One of the assumptions when using a linear regression model is that the predicted value is assumed to have been picked from a normal distribution of possible values. One of the main characteristics of a normal distribution is the observations are not too far from the mean. The normal distribution has light tail. But in practice, a perfect normal distribution is rarely true. Outliers often exist and skewness often happen. Two methods are studied when the normality is violated: the generalized linear models and robust statistics model. An example is provided to compare the results from these two models.

Joel Iiams, University of North Dakota, Jeremiah Bartz, University of North Dakota, Bruce Dearden, University of North Dakota, Jump Sizes for Polygonal Balancing Numbers

Polygonal balancing numbers are generalizations of triangular balancing numbers. Each triangular balancing number corresponds to a solution in positive integers to the Pell Equation $x^{2}-2 y^{2}=1$ and vice versa. All solutions of the Pell Equation result from multiplying a fundamental solution by a unit in $Z[\sqrt{2}]$. Thus, from an initial balancing number all others can be produced by a certain transformation. For $s$-agonal balancing numbers the companion equation is Pell-like and it is no longer true that every solution to the companion equation corresponds to an $s$-agonal balancing number. Consequently, the corresponding transformation on $s$-agonal balancing numbers must be applied a certain number of times. This is the jump size.

## David Miller (UG), Concordia College, <br> Weighted Biquandle Polynomial

Knot theory is the mathematical study of knots and a sub-field of topology. Virtual knot theory is an extension of knot theory which broadens how many objects we can study. To put it in perspective, there is only one 4 -crossing classical knot but over 500 virtual knots with 4 crossings! However, with this broadened view of knot theory, it is more needed to distinguish between two knots. Our research builds and expands upon the Generalized Alexander polynomial and $Z$-parity polynomial to create a new polynomial that is nonzero when the previous two are not.

Will Mitchell, Macalester College, Abbie Natkin, Paige Robertson, Marika Sullivan, Xuechen Yu, Chenxin Zhu

Splitting and Conformal Mapping Strategies for Nearly Singular Integrals
Gauss-Legendre quadrature and the trapezoidal rule are powerful tools for numerical integration of analytic functions. For nearly singular problems, however, these standard methods become unacceptably slow. We discuss some existing methods for improving on these schemes when the location of the singularity is known, and we introduce some new techniques. We carry out numerical tests and present an application to some nearly singular surface integrals of viscous flow.

## Oriana Penaloza (UG), Concordia College,

Ethics and Diversity in Data Science
A big problem in the technological world is the lack of diversity and representation which discourages many people from being part of the field. For this reason, this research focused on investigating how issues related to ethics and diversity can be incorporated into the Concordia DATA curriculum for future students to learn about these very important issues. The methods followed to achieve the goal of this research were to critically read literature in the field of study, and effectively communicate goals, aims, approaches, and results within the area of study through written work, datasets, and cases studies about diversity and ethics in data science for the use of future Concordia students.

## William Schwalm, University of North Dakota (retired),

Extending the Clairaut Type of Differential Equations to Higher Order
The Clairaut differential equations are first order equations of the general form

$$
y-x p=f(p),
$$

where $p=y^{\prime}(x)$. They are interesting since by differentiating the equation and factoring the result one can in principal find both the general solution and a singular solution. Finding singular solutions of first order nonlinear DEs that are polynomials in $\$ \mathrm{x}, \mathrm{y} \$$ and $\$ \mathrm{p} \$$ is not difficult. Methods of finding them for DEs of higher order are not as straightforward. Some authors have developed a theory for second order Clairaut type equations

$$
y-x p+x^{2} q / 2=f(q)
$$

where $q=y^{\prime \prime}(x)$. Here, generally, there is a family of singular solutions satisfying an auxiliary DE. These equations are discussed, together with a further extension to higher order.

## Sarah Seger, Concordia College,

Closed Seifert Surfaces and 0.5 -solvability
A slice knot was originally defined as a cross-section, or slice, of a sphere embedded in 4-dimensional space. Slice knots arise in the study of complex hypersurfaces, are related to the failure of the Whitney trick in 4 dimensions, and allow us to give the set of knots a group structure, but are difficult to detect. In 2003 Cochran, Orr, and Teichner introduced $n$-solvability. Slice knots are $n$-solvable for all $n$, and as $n$ approaches infinity we may think of $n$-solvable knots as successively finer approximations of slice knots. For knots, 0.5 -solvability is equivalent to the well-known condition of algebraic sliceness (every Seifert form has a metabolizer), but 0.5 -solvable links remain unclassified. We define specific generalizations of Seifert forms to links using the 0 -surgery manifold to obtain a necessary condition for 0.5 -solvability. Martin classifies 0 -solvable links using Milnor's invariants, however using our results we show that Milnor's invariants are insufficient to classify 0.5 -solvable links.

## James Sellers, University of Minnesota - Duluth,

On the Parity of the Number of Partitions with Odd Multiplicities
In a 2019 publication, Mike Hirschhorn and I considered the parity of the function $a(n)$ which counts the number of integer partitions of $n$ wherein each part appears with odd multiplicity. We derived an effective characterization of the parity of $a(2 m)$ based solely on properties of $m$. In this talk, we quickly prove that result, and then extend it to an explicit characterization of the parity of $a(n)$ for all $n \not \equiv$ 7 mod 8 . We also exhibit some infinite families of congruences modulo 2 which follow from these characterizations. This is joint work with Fabrizio Zanello of Michigan Technological University.

## Tom Sibley, St. John's University and College of St. Benedict (retired), <br> Colorful Linear Algebra

Convert an edge-colored graph, like a hexagon with the color $A$ on the edges, $B$ on the short diagonals and $C$ on the long diagonals to an adjacency matrix. Often Mathematica gives eigenvalues as nice linear combinations, here $A-B-C$ (twice), $C-A-B$ (also twice), $2 B-2 A-C$, and $2 A+2 B+C$. Even more surprising, the entries of the eigenvectors can be integers independent of $A, B$ and $C$. What characterizes graphs that behave so nicely? And why? I wish I knew, but I'll share what I have learned.

## Gregory Tanner, Concordia College,

Leap of Faith: Bungee Jumping Modeling Project
Students receive a request for help from an entrepreneur looking to start up a bungee jumping business, "How long and thick should a bungee jumping cord be to maximize fun while keeping the jumper safe?" This open-ended project can be tackled with second-order differential equations, energy balance, or numerical simulation. Students communicate their recommendations as a letter to the client. In this talk, we will discuss this specific modeling project and what makes a good modeling project for a differential equations course.

## Caelan Wang, University of Manitoba,

Teaching Curve Sketching with Game-Based Learning
What does game-based learning mean to you? How can we implement game-based learning in a university setting? This talk will begin by addressing these questions and then introducing a game I personally designed to help students learn and practice sketching a curve with limited information provided. A game-play experience is included in this talk for you to experience game-based learning first hand.

## Aaron Wangberg, Winona State University, Grading with a Growth Mindset

Everyone gets stuck on math problems. So who deserves more credit: The student whose solution is pristinely correct or the student demonstrating thought and struggle in their own answer? Too often, grading rewards the wrong approach. In this talk, I'll share a grading technique designed to help students develop their problem-solving abilities, promote a growth mindset toward mathematics problems, and encourage thoughtful struggle with mathematics problems.

## Matthew Wright, St. Olaf College,

Polynomial Identities for Generalized Fibonacci Sequences
Fibonacci, Pell, and Lucas numbers are known to satisfy certain polynomial identities. Thus, it seems likely that other sequences of generalized Fibonacci numbers (that is, sequences defined by second-order linear recurrences) also satisfy such identities. However, not every generalized Fibonacci sequence satisfies the type of polynomial identity in question. In this talk, I will show which families of generalized Fibonacci sequences satisfy these identities, and how these results follow from the classic Waring formula.

